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Reduction of Random Uncertainty in Differential Temperature Measurements, Using Common-Leg Thermocouples

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1. Abstract

The present work considers Type A uncertainty quantification of random error for common-leg Thermocouples (TCs) (i.e., ones that, at each TC junction inside the sensor, share a common thermoelement along their lengths). The uncertainty is presented for both a common-leg TC and for individual separate-leg TCs. For Type K TCs, an uncertainty reduction of up to 3x is possible when differential temperatures, ΔT, are above 150°C; however, this diminishes to little or no improvement at below ΔT = 150 °C. The uncertainty reduction would be different for each thermocouple type (e.g. Type S, Type R, Type B, Type N, etc.), but it is estimated it would cap out in the reduction range of 3x – 5x.

2. Introduction

Measuring the temperature of nuclear fuel is a complex endeavor [1] that requires many phenomena such as thermo- and nuclear interactions to be considered. However, first and foremost, nuclear fuel must be able to physically accommodate a sensor without disrupting the desired outcome. This drives sensors such as thermocouples (TCs) to be made smaller and more compact, and for more sensors to be built into a single probe. A common practice is to create multipoint TCs—some of which even share a common thermoelement for each TC junction inside the sensor (Figure 1). The multipoint TC is made up of a common leg and multiple opposing legs held at different locations—axially—along the length of the TC. These known positions make up the main sensing location of the thermocouple. The wires are all isolated from one another with high temperature, electrical insulators like alumina or magnesia among others. The whole build is then protected by an outer, metal sheath that protects the sensor inside from harmful chemical interactions. Finally, the sensor is directed at the cold end to a Data Acquisition System (DAS) to read out the Electromotive Force (EMF) produced by the TC in the form of millivolts. The millivolts are produced along the length of cable that is collocated with a heated zone.

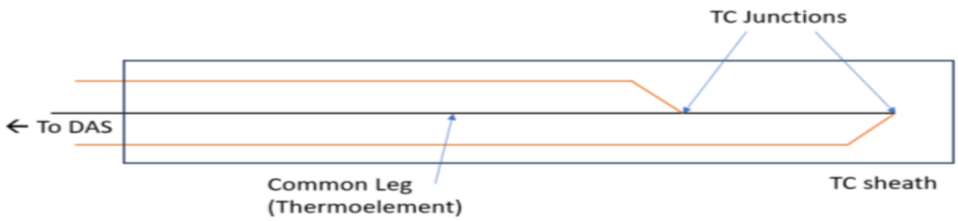

Figure 1: Wire schematic of common-leg TC with a protective outer sheath.

The temperature sensed by a TC is governed not only by the heated zone it is collocated with but also by what metals were used as thermoelements. These two governing principles provide an overall shape of the amount of EMF produced by the thermocouple. This can be seen as

$$1. \quad V_{ij} = \int_0^{L_1} S_i \frac{dT}{dx} dx + \int_{L_1}^{L_2} S_j \frac{dT}{dx} dx + \cdots + \int_{L_N}^{0} S_j \frac{dT}{dx} dx$$

where S is the material specific Seebeck coefficient, dT/dx is the local temperature profile and L is the overall length of each thermoelement. However, assuming exactly two homogeneous thermoelements, it is commonly shown in the temperature domain as follows

$$2. \quad V_{ij} = \int_{T_0}^{T} S_i dT + \int_{T}^{T_0} S_j dT = \int_{T_0}^{T} (S_A - S_B) dT$$

where all the individual wires, 1 through N, are summed up into just two distinct lengths, A and B, T is the temperature measurand under interest and T0 is the constant, reference temperature—usually 0 °C

3. Separate-Leg Thermocouples

If a differential temperature, ΔT, is desired for further analysis that extends beyond that of bulk temperature (e.g., thermal conductivity [2], heat exchanger core temperature, and heat flux [3], the difference in voltage, ΔV, can be measured and then converted to temperature through lookup tables.

Individual, separate-leg TCs that are attempting to measure ΔT across a temperature gradient will produce two different temperature values, T1 and T2, both of which are related to the same reference temperature, T0, which, as mentioned, is usually held at 0 °C. Taking Equation 2 into consideration, the form of the voltage generated by individual thermocouples can be represented as:

$$3a. \quad V_{AB} = V_A - V_B = S_A(T_1 - T_0) - S_B(T_1 - T_0)$$

$$3b. \quad V_{CD} = V_C - V_D = S_C(T_2 - T_0) - S_D(T_2 - T_0)$$

with each equation showing the EMF generated between any two different metals. The differential voltage, ΔV, between two separate leg TCs is then calculated as:

$$4. \ \Delta V_{SL} = V_{AB} - V_{CD}$$

with ΔVSL representing the differential voltage between two separate leg thermocouples and subscripts A, B, C, and D representing the 4 unique thermoelements with their unique material properties utilized as TC wire.

4. Common-Leg Thermocouples

For common-leg TCs, as seen in Figure 1, Equation 2 is utilized as in separate leg TCs, but, by sharing the material properties of one of the legs, A, the final result is slightly different

$$5a. \ V_{AB} = V_A - V_B = S_A(T_1 - T_0) - S_B(T_1 - T_0)$$

and

$$5b. \ V_{AD} = V_A - V_D = S_A(T_2 - T_0) - S_D(T_2 - T_0)$$

Combining the two equations, 5a and 5b, gives the differential EMF generated between two TCs sharing a common leg:

$$6. \ \Delta V_{CL} = V_{AB} - V_{AD}$$

5. Uncertainty Quantification

An analysis of Type A uncertainty quantification is herein shown for the ideal case of four TC junctions: two individual TCs and two that share a common leg between them. These four TCs are mathematically superimposed, in both space and time, in place of one another so as to compare the reduction of any uncertainty between the two TCs. Only random errors are being considered here. Various techniques can be employed to reduce the standard error to a minimum.

For the separate-leg TCs, the uncertainty quantification, U, follows a standardized equation [5], but herein applied specifically to Equation 4:

$$7.\ U_{\Delta V}(V_{AB}, V_{CD}) = 2\sqrt{\left(\frac{d\Delta V}{dV_{AB}}\sigma_{AB}\right)^2 + \left(\frac{d\Delta V}{dV_{CD}}\sigma_{CD}\right)^2}$$

where σ is the random error, which is usually the sample standard deviation. The leading 2 brings the uncertainty to the 95% confidence interval. However, for the common-leg TC, the temperature measurand has correlation between any two measurands, driving the covariance term to be added:

$$8.\ U_{\Delta V}(V_{AB}, V_{CD}) = 2\sqrt{\sum_{i=1}^{N}\left(\frac{d\Delta V}{dV_i}\sigma_i\right)^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{df}{dx_i}\frac{df}{dx_j}\sigma_{ij}}$$

where σ12 is the covariance, defined as:

$$9.\ \sigma_{ij} = \frac{1}{N}\sum_{i,j=1}^{N}\left[(x_i - \bar{x}_i)(x_j - \bar{x}_j)\right] = \rho_{ij}\sigma_i\sigma_j$$

where $\bar{x}$i,j is the time-averaged mean of the overall measurements of a single TC, ρij is the correlation term between two distinct thermocouples, and σi,j are the standard deviations of individual thermocouple measurements, respectively.

5.1. Uncertainty Quantification of ΔV, without Correlated Error

Applying Equation 7 to Equation 4 for separate-leg TCs that are measuring a ΔV—assuming low to zero correlation between measurands—gives:

$$10a.\ \frac{d\Delta V_{SL}}{dV_{AB}} = 1$$

$$10b.\ \frac{d\Delta V_{SL}}{dV_{CD}} = -1$$

This means the general form for representing the random uncertainty of separate-leg TCs is:

$$11.\ U_{SL} = 2\sqrt{{b_{AB}}^2 + {b_{CD}}^2}$$

Note the negative value of Eq. 10b. This will play a significant role in the next section.

5.2. Uncertainty Quantification of ΔV, with Correlated Error

For random correlated error from a common-leg TC (see Equation 6), the random uncertainty includes the covariance term, as in Equation 8, therefore the covariance term in Equation 12 is an overall reduction in uncertainty, as the form factor of Equation 6 gives a leading negative for the covariance term from something similar to Equation 10b but instead is $d\Delta V_{CL}/dV_{AD}$=-1. The total uncertainty at the 95% level is therefore:

$$12.\ U_{CL} = 2\sqrt{{b_{AB}}^2 + {b_{AD}}^2 - 2\rho_{ABAD}\sigma_{AB}\sigma_{AD}}$$

where the higher the correlation the less the uncertainty in ΔV of Eq. 6.

5.3. Reduction Factor

With the covariance term being negative, as seen in Equation 12, use of a common-leg TC can reduce the overall random uncertainty per:

$$13.\ U_{CL} = \frac{U_{SL}}{R}$$

where R is a unitless number. Different TC materials may greatly affect R, but it is estimated that R is bounded by 1, up to a finite value of around 3–5.

6. Experimental Results

Repeated tests were run using Type K exposed junction TCs. A common-leg TC with two junctions as well as two individual, separate-leg TCs were utilized in an isothermal, ΔT measurement. Each TC was isolated from the others by using a DAS with over

240 Vrms channel-to-channel isolation (or 60 VDC). The two leading TC junctions were exposed to elevated temperatures of ~350 °C, then the secondary TC junction was exposed to varying temperatures, in stages, ranging from room temperature up to ~340 °C—producing overall ΔT measurements of ~10 °C–300 °C.

(Figure 2) shows that, when using common-leg TCs to measure smaller ΔT values (under 150 °C), uncertainty can be reduced by up to 3x in comparison to using separate-leg TCs. However, when ΔT is large (over 150 °C), the reduction factor diminishes to unity, meaning it would not matter which method was utilized.

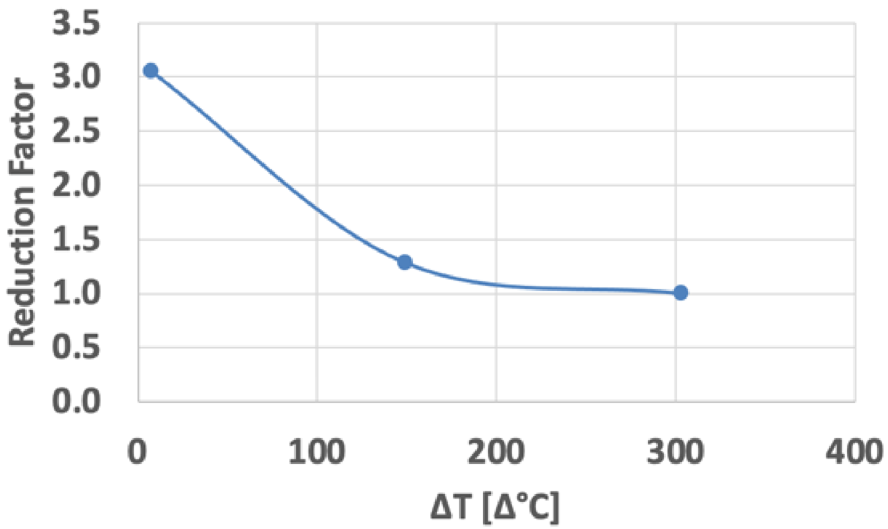

Figure 2. Reduction factor when using common-leg TCs vs. two individual TCs to measure ΔT. The chart shows that, with expanding ΔT, diminishing returns are gained from using common leg TCs.

7. Conclusion

Due to the form factor of the ΔV equation (Equation 4 and Equation 6), the uncertainty can be reduced by up to 3x when measuring ΔT values that are close together in magnitude. This is important, as ΔT can be utilized in further calculations (e.g., heat flux and thermal conductivity), and reducing the uncertainty in the random error in turn reduces the amount propagated on to the later calculations.

References

1. Palmer, J., et al. 2021. "Summary of Thermocouple Performance in the Advanced Gas Reactor Experiment AGR-5/6/7 during Irradiation in the Advanced Test Reactor." Idaho National Laboratory, INL/CON-21-63280-Rev000.
2. Bird, R. B. 2002. "Transport phenomena." Appl. Mech. Rev., 55(1), R1-R4.
3. Fuchs, M. 1986. "Heat Flux." Methods of Soil Analysis: Part 1 Physical and Mineralogical Methods. 957-968.
4. Kasap, S. 2001. "Thermoelectric effects in metals: Thermocouples." Canada: Department of Electrical Engineering University of Saskatchewan.
5. PTC, A. 2018. PTC 19.1-2018 (Revision of ASME PTC 19.1-2013). American Society of Mechanical Engineers.